Design of Desirable Handling Qualities for Aircraft Lateral Dynamics

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The model-matching algorithm proposed by Curran is discussed, and the technique is applied to the design of desirable lateral handling qualities. This algorithm yields a state feedback law with dynamic compensation. The design specifications are in the form of lateral-directional handling qualities criteria. The transfer function of the decoupled desirable model, which consists of the bank and sideslip angles to the aileron and rudder input, is selected according to the criteria. Two analytical control laws are synthesized using the algorithm and are based on the simplified dynamics of an aircraft. Different types of aircraft with unacceptable handling qualities in the unaugmented condition are considered as examples. The results of simulation studies that were performed to illustrate as well as to compare the two control laws are given.

Nomenclature

(A,B,C) = coefficient matrices of the plant in Eq. (1)
A_ϕ = leading coefficient of the numerator quadratic in the $\phi(s)/\delta_a(s)$ transfer function
E = $m \times m\rho$ matrix defined in Eq. (A1)
(F,G,H) = coefficient matrices of the desirable model in Eq. (2)
f_{ij} = parameters defined in Eq. (10)
g = acceleration of gravity
I = unit matrix of appropriate size
I = $m \times m$ unit matrix
J_{ij} = auxiliary matrices to make a system equicontrollable
(K_ϕ, K_β) = sensitivities of control inputs in Eq. (6)
L_i = $m \times m$ matrices defined in Eq. (B6)
$(L_{(\ )}, N_{(\ )}, Y_{(\ )})$ = dimensional stability derivatives in Eq. (4)
(M,N,P,Q,R,S) = gain matrices defined in Eq. (B13)
m = number of inputs or outputs
n = state dimension of the plant
p = roll rate, or state dimension of the model
R^n = Euclidian n-dimensional space
r = yaw rate
s = Laplace operator
T = transformation matrix defined in Eq. (A3)
u = m-dimensional input vector
V_0 = flight velocity
$W(s), W(A,B,C)$ = transfer function, $C(sI-A)^{-1}B$
x = state vector
y = m-dimensional output vector
z = auxiliary state vector
α = $-\alpha$ denotes the pole location of z
β = sideslip angle or modification parameter in Eq. (B6)
γ_0 = glide path angle
δ = control surface deflection
ζ = damping ratio
θ = $m \times m$ zero matrix
(λ,μ,ν) = parameters defined in Eq. (B6)
ρ = controllability index
τ = time constant
ϕ = bank angle
ω = natural frequency

Superscripts

T = matrix or vector transpose
-1 = inverse of a matrix
$(\bar{\ })$ = augmented vector or matrix in step 2
$(\tilde{\ })$ = transformed vector or matrix of $(\bar{\ })$ in step 3
$(\hat{\ })$ = augmented vector or matrix in step 4
$(\hat{\ })^*$ = transformed vector or matrix of $(\hat{\ })$ in step 5

Subscripts

a = aileron
d = dutch roll
L = Luenberger's form
m = model
p = plant or roll rate
r = yaw rate, rudder, or roll mode
s = spiral mode
β = sideslip angle
ϕ = bank angle

$(\ )(s,\rho)$ $= \sum_{i=1}^{\rho} (\ )_i s^{i-1}$

Presented as Paper 77-1045 at the AIAA Guidance and Control Conference, Hollywood, Fla., Aug. 8-10, 1977; submitted Sept. 30, 1977; revision received April 18, 1978.

Index category: Handling Qualities, Stability and Control.

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I. Introduction

DURING the past decade, a number of methods have been proposed for the design of stability augmentation systems (SAS) for aircraft using modern control theory.[1-9]

One of the most successful of these is the linear regulator method,[1,2] which has been combined with the perfect model-following method of Erzberger[3] by Asseo[4] and by Winsor and Roy.[5] A disadvantage of the perfect model-following method is that it can be applied only to a limited class of controlled plants. To overcome some of the disadvantages of optimal control, Rediess and Whitaker[6] introduced a new performance index, the model PI, and a parameter optimization design procedure that starts with some practical engineering specifications and uses the model PI as a synthesis tool so as to obtain a satisfactory design.

Another useful design technique that has been developed recently is the model-matching method.[10-12] Wolovich and Shirley[7] worked on the handling qualities of a hovering helicopter, but they had to compromise substantially the design model so as to obtain a solution. They had to do this because their method did not permit the changing of the numerator zeros of the transfer functions. On the other hand, by combing linear system theory and differential calculus, Montgomery and Hatch[8] developed a procedure that allows the direct calculation of the required sets of the feedback and feedforward gains.

The purpose of this paper is to derive an analytical control law that can be used to establish a specific pole-zero configuration for a system that can provide desirable lateral handling qualities for an aircraft. Some previous work already has been done by one of the authors, Ohta,[9] who investigated the design of desirable longitudinal handling qualities and proposed some analytical solutions. This paper extends this previous work. In it, a description is first given of the model-matching algorithm that is proposed in Ref. 12. This algorithm yields a state feedback law with dynamic compensation. In this algorithm, the specification of the numerator zeros is realized by dynamic compensation, which is essential if a high-performance aircraft is to exhibit desirable handling characteristics. The determination of the analytical control laws that will yield the desirable handling qualities related to the lateral response of an aircraft is discussed next. A desirable model is selected to represent the criteria, starting with the design specifications. Stability augmentation systems then are synthesized using the algorithm described. Finally, the design is substantiated by numerical simulations.

II. Model-Matching Method

A main requirement of a model-matching method is that of finding a controller that, when applied to the plant, causes it to behave, in an input-output sense, in the same way as the desirable model. Wolovich[10] and Moore and Silverman[11] derived the necessary and sufficient conditions for the existence of solutions using different approaches. Wolovich discusses the problem in the frequency domain and describes a design method that results in a static feedback law. On the other hand, Moore and Silverman describe a design method that is based on a structural algorithm in the time domain and does not utilize any coordinate transformation. Their method can yield a solution with dynamic compensation, but the solution is not in a state feedback form.

The design that is described in this paper is based on the work originally suggested by Curran.[12] He discusses the problem from the viewpoint of model following. Using a state equation in the equicontrollable canonical form, he derives a design algorithm that yields a solution of a state feedback law with dynamic compensation. His method may be said to be an extension of Ref. 10. The algorithm permits the zeros of the numerator in the transfer function of the plant to be placed at the desirable location. This is essential if a high-performance aircraft is to exhibit desirable handling characteristics. However, as in the classical control, locations of unstable zeros of the plant cannot be changed by this algorithm, because of the stability requirement of the compensator.

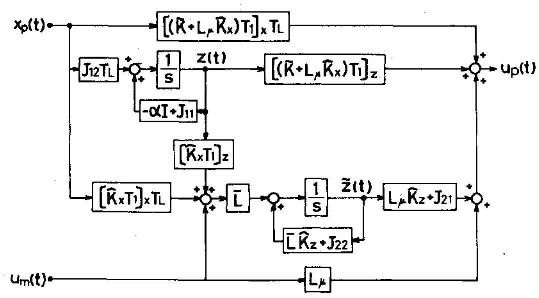

Fig. 1 Block diagram of the model-matching controller.

Let the linear, time-invaviant system that is to be controlled and the model that is to be specified be described, respectively, by the state equations

$$\dot{x}_p = Ax_p + Bu_p \qquad y_p = Cx_p \tag{1}$$

$$\dot{x}_m = Fx_m + Gu_m \qquad y_m = Hx_m \tag{2}$$

where $x_p \in R^n$, $x_m \in R^p$, u_p, u_m, y_p, and $y_m \in R^m$. Furthermore, let it be assumed that all of the matrices in Eqs. (1) and (2) have appropriate sizes, that matrices B, C, G, and H have full rank, and that both the plant and model are completely controllable and observable.

Equicontrollability is defined in Ref. 12 in the following way: the pair (A, B) with controllability index ρ is equicontrollable if $n = m\rho$, where $A \in R^{n\times n}$ and $B \in R^{n\times m}$. A system that satisfies this definition is called an equicontrollable system. If a system (A, B, C) is equicontrollable, it can be transformed into the equicontrollable canonical form $(\tilde{A}, \tilde{B}, \tilde{C})$:

$$\tilde{A} = \begin{bmatrix} 0 & I & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & I \\ \tilde{A}_1 & \tilde{A}_2 & \dots & \tilde{A}_\rho \end{bmatrix}$$

$$\tilde{B} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ I \end{bmatrix} \qquad \tilde{C} = [\tilde{C}_1 \quad \tilde{C}_2 \quad \dots \quad \tilde{C}_\rho] \tag{3}$$

where all of the elements in $(\tilde{A}, \tilde{B}, \tilde{C})$ are $m \times m$. The transformation matrix is given in Eq. (A3) of Appendix A. This form is a direct extension to multi-input systems of the single-input phase-variable canonical form of Ref. 13.

The model-matching algorithm used in this paper is described in Appendix B. The algorithm always provides a feedback law when the plant and model have inputs and outputs of equal number. There are six steps in the algorithm, outlined below:

Step 1) Transform the plant and model into Luenberger's controllable canonical form,[14] and check the equicontrollability of them.

Step 2) Luenberger's form is used for augmenting the states of the plant and model so as to place them in equicontrollable systems with the same state dimension. The state z in Eq. (B1) contributes to this augmentation.

Step 3) The plant and model are transformed into the equicontrollable canonical form, and a loop of feedback $\tilde{K}$ in Eqs. (B3) is applied to the plant in order to place all of its eigenvalues at the origin.

Step 4) A dynamic compensator $\tilde{z}$ in Eq. (B8) is so synthesized that the numerator of the plant may coincide with that of the model. The realizability of the compensator is also examined in this step.

Step 5) Find the feedback matrix $\hat{K}^*$ in Eq. (B10) which places all of the poles of the plant to the desirable location.

Step 6) Transform the compensator so far derived into the original coordinate, and find a minimal realization of it. The resulting control law is given in Eqs. (B12) and is illustrated in Fig. 1.

III. Description of Aircraft Dynamics

The linearized lateral equations of motion of the aircraft are given in Ref. 15 and are represented by Eq. (1), where x_p, u_p, and y_p are expressed as

$$x_p^T = [p \;\; \phi \;\; r \;\; \beta] \qquad u_p^T = [\delta_a \;\; \delta_r] \qquad y_p^T = [\phi \;\; \beta]$$

The coefficient matrices $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{C}$ are given by

$$A = \begin{bmatrix} L_p' & 0 & L_r' & L_\beta' \\ 1 & 0 & 0 & 0 \\ N_p' & 0 & N_r' & N_\beta' \\ Y_p^* & Y_\phi^* & Y_r^* - 1 & Y_\beta \end{bmatrix} \quad B = \begin{bmatrix} L_{\delta a}' & L_{\delta r}' \\ 0 & 0 \\ N_{\delta a}' & N_{\delta r}' \\ Y_{\delta a}^* & Y_{\delta r}^* \end{bmatrix} \tag{4}$$

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

So-called primed stability derivatives are used in Eqs. (4), and $(\;)_\beta' \triangleq (\;)_v' V_0$, $Y_i^* \triangleq Y_i / V_0$, and $Y_\phi^* \triangleq g \cos\gamma_0 / V_0$. However, primes and asterisks are dropped, for simplicity of the nomenclature, in the following.

The flight conditions and values of the stability derivatives used in numerical examples of Sec. VI are given in Tables 1 and 2 of Ref. 16. Aircraft 1 is a conventional small jet airplane during cruise, and aircraft 2 is a STOL airplane during steep descent. These different types of aircraft are taken from Refs. 15 and 17, respectively, and are used to substantiate the control laws derived in Sec. V.

By using a digital computer, the model-matching algorithm explained in Sec. II can yield a solution without any simplification of the aircraft dynamics in Eqs. (4). The solution, however, is effective only to a specific flight condition and requires a high-order dynamic compensator in order to change the zeros of the plant. This is not adequate from a practical point of view. Instead, in this paper, a closed-form solution will be sought at the cost of simplification of the lateral dynamics. For these reasons, the stability derivatives Y_p, Y_r, and $Y_{\delta a}$, which are usually small, are neglected in Eq. (4). Moreover, according to the three-degree-of-freedom dutch roll approximation, the derivatives L_r and N_p are assumed to be neglected. This simplification of the dynamics leads to the derivation of simpler analytical control laws. The adequacy of simplification is discussed in Sec. VI.

IV. Selection of a Desirable Model

The specific lateral-directional handling qualities criteria that are selected for this study are shown in Fig. 2 and are based on the information given in Refs. 6 and 18. These criteria consist of specified regions in the s-plane for the roll mode and dutch roll mode poles and the numerator-to-denominator frequency ratio of the dutch roll mode corresponding to good, satisfactory, and acceptable handling qualities. One of the important indicators of the quality of the lateral response is the bank angle to aileron transfer function.

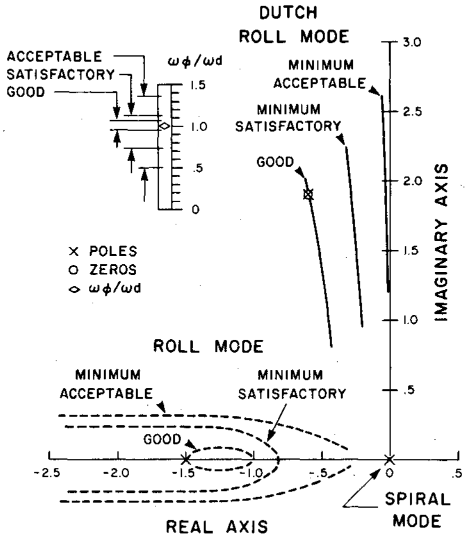

Fig. 2 Lateral-directional handling qualities.

The criteria have shown that seven handling parameters in the transfer function

$$\frac{\phi(s)}{\delta_a(s)} = \frac{A_\phi[(s/\omega_\phi)^2 + 2\zeta_\phi(s/\omega_\phi) + 1]}{\underbrace{(\tau_s s + 1)}_{\text{spiral}}\underbrace{(\tau_r s + 1)}_{\text{roll subsidence}}\underbrace{[(s/\omega_d)^2 + 2\zeta_d(s/\omega_d) + 1]}_{\text{dutch roll}}} \tag{5}$$

influence a pilot's opinion of the lateral response. The desirable numerical values of the handling qualities parameters used in this study, as well as those of the basic aircraft, are listed in Table 1. The locations of the desirable poles and zeros for the first aircraft are superimposed on Fig. 2. This figure and table show that the aircraft, for this flight condition and without SAS, would have an unacceptable dutch roll. One way to eliminate an undesirable dutch roll is by pole-zero cancellation in Eq. (5). The resulting transfer function contains the spiral and roll subsidence poles, which are placed in the desirable region in Fig. 2.

On the other hand, it is necessary to have a model of some transfer characteristics in which the dutch roll mode is dominant. A suitable one is the sideslip angle to rudder transfer function, $\beta(s)/\delta_r(s)$. It is only necessary to select zeros to establish the model. The handling qualities criteria do not place any restrictions on these zeros, but the dutch roll mode should be dominant. One zero is essentially at the origin, and one is typically near the roll mode pole. Hence, it is necessary for these zeros to cancel the spiral and roll subsidence modes.

Summarizing the preceding discussion, the decoupled transfer function of the desirable model becomes

$$W_m(s) = \begin{bmatrix} \dfrac{\phi_m(s)}{\delta_{am}(s)} & 0 \\ 0 & \dfrac{\beta_m(s)}{\delta_{rm}(s)} \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{K_\phi}{[s + (1/\tau_s)][s + (1/\tau_r)]} & 0 \\ 0 & \dfrac{K_\beta}{(s^2 + 2\zeta_d\omega_d s + \omega_d^2)} \end{bmatrix} \tag{6}$$

Table 1 Handling qualities parameters for the $\phi(s)$ transfer function

Handling qualities parameters	Aircraft 1		Aircraft 2	
	Basic	Desired	Basic	Desired
$1/\tau_s$	−0.0014	0.01	0.03	0.01
$1/\tau_r$	1.777	1.5	2.654	6.0
ζ_d	0.024	0.3	0.0029	0.606
ω_d	1.878	2.0	1.308	2.116
ζ_ϕ	0.0047	0.3	0.606	0.606
ω_ϕ	1.589	2.0	2.116	2.116
ω_ϕ/ω_d	0.846	1.0	1.618	1.0

Four poles of the model can be selected freely within the regions corresponding to good handling qualities in Fig. 2. Complex zeroes are selected to cancel the dutch roll mode poles in $\phi_m(s)/\delta_{am}(s)$, and the dutch roll mode is dominant in $\beta_m(s)/\delta_{rm}(s)$.

V. Analytical Control Laws for Lateral SAS

In Sec. III, the stability derivatives Y_p, Y_r, $Y_{\delta a}$, L_r, and N_p were neglected in Eqs. (4). However, it is difficult to derive an analytical control law using the model-matching algorithm. Candidates for further simplification are $L_{\delta r}$ and $Y_{\delta r}$. They cannot, however, be neglected simultaneously because of the controllability requirement imposed on the plant. Hence, the first derivation of the control law is based on the simplified dynamics of aircraft neglecting $L_{\delta r}$, and the second one is based on the dynamics neglecting $Y_{\delta r}$.

Control Law 1 (Assume $L_{\delta r}=0$)

In this case, model matching can be attained by feedback compensation with one dynamic state $\tilde{z}$. The control law is given by

$$u_p = N\tilde{z} + Qx_p + L_\mu u_m \tag{7a}$$

$$\dot{\tilde{z}} = M\tilde{z} + Px_p + \bar{L}u_m \tag{7b}$$

where $u_m^T = [\delta_{am}, \delta_{rm}]$ represents the reference aileron and rudder control inputs to the augmented plant, respectively. The gain matrices in Eqs. (7) are defined by

$$N = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad Q = \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} \qquad L_\mu = \frac{K_\phi}{L_{\delta a}} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$M = K_6 - f_{22} \qquad \bar{L} = (1/Y_{\delta r})\,[K_\phi K_4/K_5,\ \ K_\beta]$$

$$Q_1 = -(1/L_{\delta a})\,[L_p + f_{21},\ f_{11},\ 0,\ L_\beta]$$

$$Q_2^T = -\frac{1}{K_5}\begin{bmatrix} N_{\delta a}L_pK_2 + L_{\delta a}N_{\delta r}Y_\phi \\ Y_\phi K_3 \\ -L_{\delta a}N_rK_2 - K_3 \\ Y_\beta K_5/Y_{\delta r} + K_3K_6 \end{bmatrix} \tag{8}$$

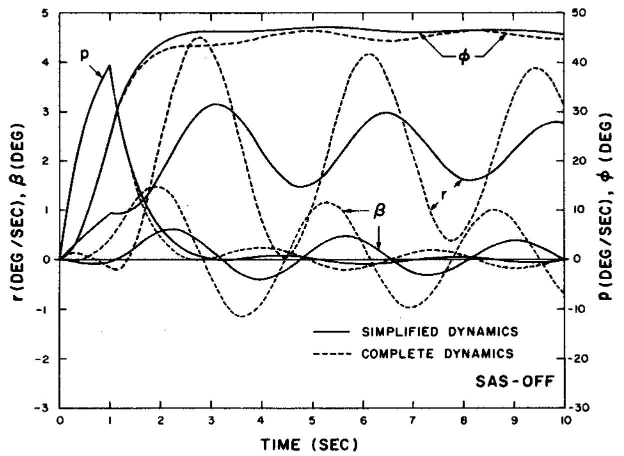

Fig. 3a SAS-off responses of aircraft 1 to a 3-deg impulse aileron input.

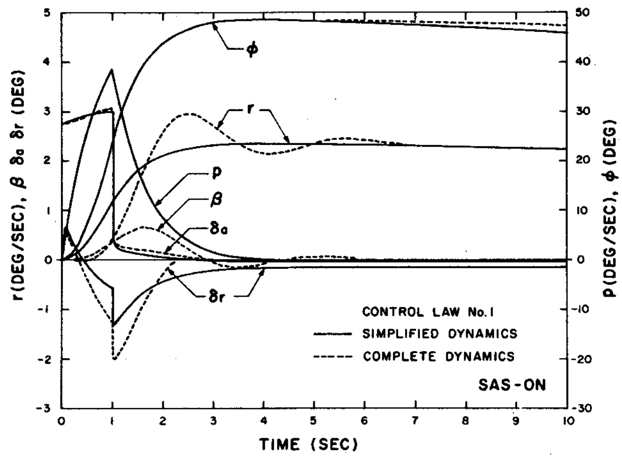

Fig. 4a SAS-on responses of aircraft 1 to a 3-deg impulse aileron input.

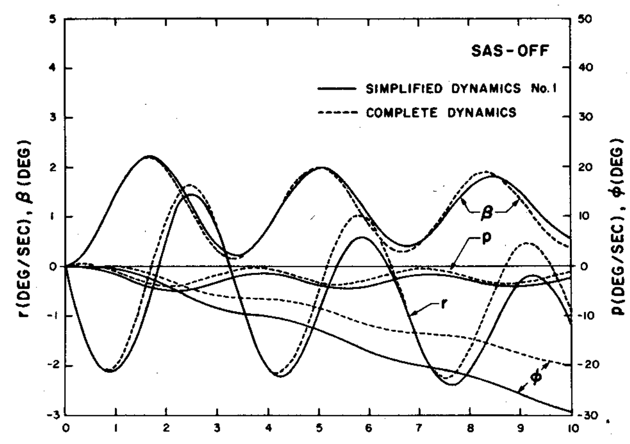

Fig. 3b SAS-off responses of aircraft 1 to a 3-deg step rudder input.

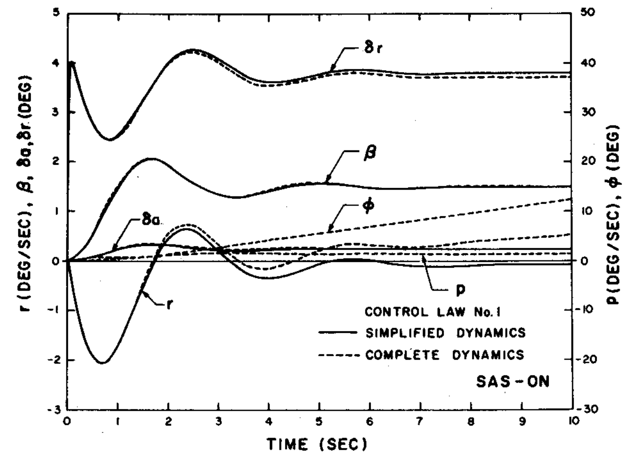

Fig. 4b SAS-on responses of aircraft 1 to a 3-deg step rudder input.

$$P^T=\frac{K_6}{K_5}\begin{bmatrix} N_{\delta a}f_{22}K_2+(K_4/K_2)(f_{22}-f_{21}) \\ -N_{\delta a}f_{11}K_1+L_{\delta a}L_{\delta r}Y_\phi(f_{22}-f_{11}/K_6) \\ -L_{\delta a}f_{22}K_2 \\ L_{\delta a}N_{\delta r}f_{12}+K_3(f_{22}-f_{12}/K_6) \end{bmatrix}$$

where

$$K_1=Y_{\delta r}(L_p-N_r)-N_{\delta r}$$
$$K_2=Y_{\delta r}N_r+N_{\delta r}$$
$$K_3=N_{\delta a}Y_{\delta r}L_\beta-L_{\delta a}Y_{\delta r}N_\beta+L_{\delta a}N_{\delta r}Y_\beta$$
$$K_4=N_{\delta a}K_1K_2+L_{\delta a}N_{\delta r}Y_{\delta r}Y_\phi$$
$$K_5=Y_{\delta r}K_3-L_{\delta a}N_{\delta r}K_2$$
$$K_6=K_2/Y_{\delta r} \tag{9}$$

The coefficient f_{ij} in Eqs. (8) are determined by the desirable location of the poles and are defined by

$$f_{11}=\frac{1}{\tau_s\tau_r},\quad f_{21}=\frac{1}{\tau_s}+\frac{1}{\tau_r},\quad f_{12}=\omega_d^2,\quad f_{22}=2\zeta_d\omega_d \tag{10}$$

Control Law 2 (Assume $Y_{\delta r}=0$)

In this case, model matching can be attained only by a feedback compensation, whose control law is given by

$$u_p=Qx_p+L_\mu u_m \tag{11}$$

where the gain matrices Q and L_μ are given by

$$Q=\tilde{Q}+L_\mu\hat{Q}\qquad \Delta=L_{\delta a}N_{\delta r}-N_{\delta a}L_{\delta r}\qquad L_\mu=\frac{1}{\Delta}\begin{bmatrix} K_\phi N_{\delta r} & K_\beta L_{\delta r} \\ -K_\phi N_{\delta a} & -K_\beta L_{\delta a}\end{bmatrix} \tag{12a}$$

$$\tilde{Q}=\frac{1}{\Delta}\begin{bmatrix} -(N_{\delta r}L_p+L_{\delta r}Y_\phi),-L_{\delta r}Y_\phi Y_\beta,L_{\delta r}(N_r+Y_\beta),-(N_{\delta r}L_\beta-L_{\delta r}N_\beta+L_{\delta r}Y_\beta^2) \\ (N_{\delta a}L_p+L_{\delta a}Y_\phi),L_{\delta a}Y_\phi Y_\beta,-L_{\delta a}(N_r+Y_\beta),(N_{\delta a}L_\beta-L_{\delta a}N_\beta+L_{\delta a}Y_\beta^2)\end{bmatrix} \tag{12b}$$

$$\hat{Q}=\frac{1}{\Delta}\begin{bmatrix} -N_{\delta r}f_{21},-(N_{\delta r}f_{11}+L_{\delta r}Y_\phi f_{21}),L_{\delta r}f_{21},-L_{\delta r}(f_{11}+Y_\beta f_{21}) \\ N_{\delta a}f_{22},\ (N_{\delta a}f_{12}+L_{\delta a}Y_\phi f_{22}),-L_{\delta a}f_{22},L_{\delta a}(f_{12}+Y_\beta f_{22})\end{bmatrix} \tag{12c}$$

The lateral decoupled control laws, Eqs. (7) and (11), are particularly interesting because they are derived analytically. There is no need for this rederivation, and they can be utilized for different flight conditions and for different types of aircraft. Both of the control laws require full state feedback of the plant for their implementation. These states, p, ϕ, r, and β, are measured by roll rate, vertical, and yaw rate gyros, and sideslip sensor, respectively.

VI. Simulation Studies

To substantiate the validity of the control laws, simulation studies were conducted for the two types of aircraft mentioned previously. Numerical values of the stability derivatives are given in Table 2 of Ref. 16. The desirable locations of the poles and zeros of the model are shown in Table 1. They were selected according to the criteria given in Fig. 2 and the recommendations found in Ref. 17.

Example 1: Aircraft 1

The responses of the unaugmented and augmented aircraft to a 3-deg rectangular pulse aileron input δ_{am} of 1 s duration (the term "impulse" instead of "rectangular pulse of 1 s duration" is used hereafter) and a 3-deg step rudder input δ_{rm} are shown in Figs. 3-5. To illustrate the effect of simplifying the aircraft dynamics, two kinds of time histories, using both simplified and complete dynamics, are presented in these figures. Figure 3 shows that the corresponding responses of the state variables between the complete dynamics and the simplified dynamics are similar. Only the amplitudes of r and β are different. It is to be noted that two kinds of simplified dynamics ($L_{\delta r}=0$ for simplification 1 and $Y_{\delta r}=0$ for simplification 2) produce the same unaugmented responses for a δ_a input. For a δ_r input, the unaugmented responses of these simplified dynamics are different. It is confirmed that in Fig. 3b the responses p and ϕ for the simplified dynamics 2 are almost the same as those of the complete dynamics, and the response β is the same as that of the simplified dynamics 1. The response of r is between the complete and simplified dynamics 1. These responses for the simplified dynamics 2 are not shown in Fig. 3b.

Figures 3a and 3b also illustrate the problems caused by the underdamped dutch roll mode, induced sideslip, and turn coordination. That these poor characteristics can be improved using SAS is shown in Fig. 4 for control law 1 and in Fig. 5 for control law 2. The sensitivities selected, K_ϕ and K_β, are such that the model would produce almost the same rolling and yawing moments due to aileron and rudder control surface deflections as those of the unaugmented airplane. The gain values in Eqs. (8) and (12) are given in Table 2. The results shown in Figs. 4a and 5a confirm that the response of the bank angle ϕ can be matched exactly with that of the model, while the undesirable β response could be almost eliminated and decoupled from the δ_{am} input. Figures 4b and 5b also illustrate that the response of the sideslip angle β can be matched with that of the model, and the bank angle ϕ can be decoupled from the δ_{rm} input. It is seen in these figures that control law 1 reveals better decoupling performance for the δ_{am} input, whereas control law 2 is better for the δ_{rm} input.

Example 2: Aircraft 2

Simulation studies also were carried out using a STOL airplane to illustrate the effectiveness of the control laws for an unconventional airplane . Since the control laws resulted in a good performance for the STOL airplane during the high-speed operation, the steep descent condition was selected for further simulation studies.

Figures 6a and 6b show the unaugmented responses for 3-deg step aileron and rudder inputs, respectively. It is seen in these figures that the dynamics of the STOL airplane are oversimplified for this flight condition, and that there are great differences in the responses between the complete and the simplified dynamics. However, as can be seen in Fig. 7, both of the proposed control laws yield good augmented responses for the complete dynamics, and the decoupling performance between the two command inputs almost is attained.

The initial condition of the auxiliary state $\bar{z}$ in control law 1 was chosen to be zero throughout these simulations. The closed-loop system using control law 1 exhibits the characteristics of a high-order system with the auxiliary state $\bar{z}$. References 19 and 20 demonstrate the degrading effect of these additional dynamics. As stated in Sec. III, this is a reason why the simplification of aircraft dynamics was made

Table 2 Gain values of the control laws for example aircraft

Control law	Aircraft 1 ($K_\phi = 25.0, K_\beta = 2.0$)				
1	$Q = \Big[$	6.929×10^{-3}	-5.499×10^{-4}	0	1.667×10^{-1} $\Big]$
		-1.769×10^{-2}	1.915×10^{-3}	-1.049×10^{-1}	2.529
	$L_\mu = [$	9.166×10^{-1}, [a]		$M = -1.187 \times 10^{2}\,]$	
	$P = [$	-1.894	-5.064	1.036×10^{2}	$-3.393 \times 10^{2}\,]$
	$\bar{L} = [$	3.171×10^{1}	$1.724 \times 10^{2}\,]$		
2	$Q = \Big[$	1.293×10^{-2}	3.091×10^{-3}	-4.552×10^{-2}	1.998×10^{-1} $\Big]$
		-5.067×10^{-2}	-1.209×10^{-1}	1.146	-1.644
	$L_\mu = \Big[$	9.110×10^{-1}	-3.081×10^{-2} $\Big]$		
		2.643×10^{-1}	1.460		

Control law	Aircraft 2 ($K_\phi = -2.845, K_\beta = 0.9633$)				
1	$Q = \Big[$	1.220	2.078×10^{-2}	0	-4.212×10^{-1} $\Big]$
		-4.445×10^{-1}	1.486×10^{-2}	-7.358×10^{-1}	1.615
	$L_\mu = [$	9.856×10^{-1}, [a]		$M = -3.427 \times 10^{1}\,]$	
	$P = [$	8.553	-2.536×10^{1}	8.331×10^{1}	$-1.383 \times 10^{2}\,]$
	$\bar{L} = [$	3.510	$3.059 \times 10^{1}\,]$		
2	$Q = \Big[$	1.220	-2.156×10^{-1}	7.655×10^{-1}	-6.292×10^{-1} $\Big]$
		2.505×10^{-3}	-5.832×10^{-1}	1.952	-2.746
	$L_\mu = \Big[$	1.0	9.449×10^{-2} $\Big]$		
		1.526×10^{-1}	1.0		

[a] 1 – 1th element.

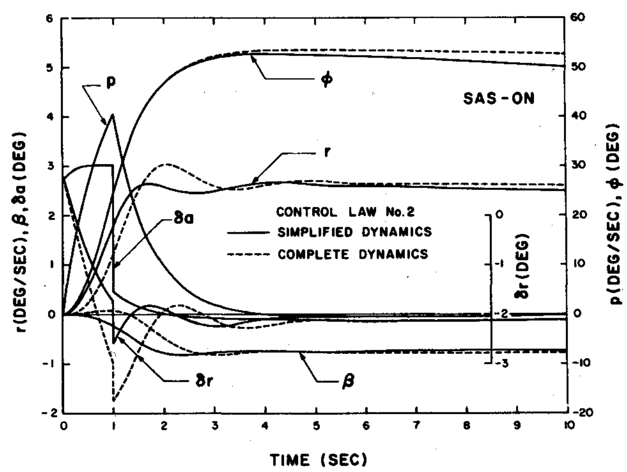

Fig. 5a SAS-on responses of aircraft 1 to a 3-deg impulse aileron input.

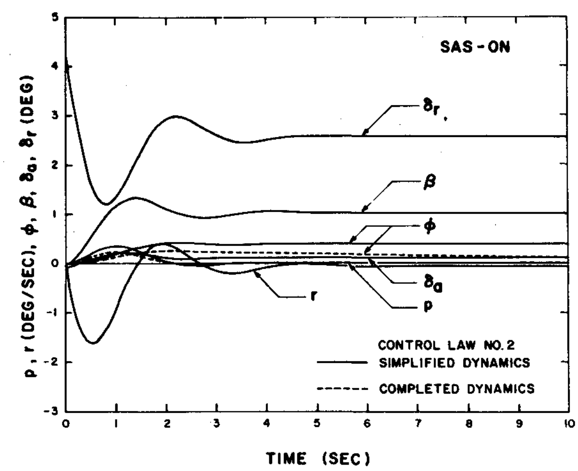

Fig. 5b SAS-on responses of aircraft 1 to a 3-deg step rudder input.

and the use of high-order auxiliary dynamics was avoided. It was confirmed in these simulations that the state $\tilde{z}$ decreased rapidly to the steady state, zero, after excitation. The strong stability characteristics of $\tilde{z}$ also are expected from a comparatively big negative value of M in Table 2. It is to be noted that the magnitudes of the control inputs are within the allowable limit in these results.

Control law 2 can be implemented using the same measurement signals as in Refs. 2-5, where linear-quadratic model-following methods are used. Control law 1 requires one additional dynamic state $\tilde{z}$ and is more complex than control law 2. The implementation, however, would be feasible on a computer. Moreover, the results are suitable for gain scheduling because they are given in closed forms.

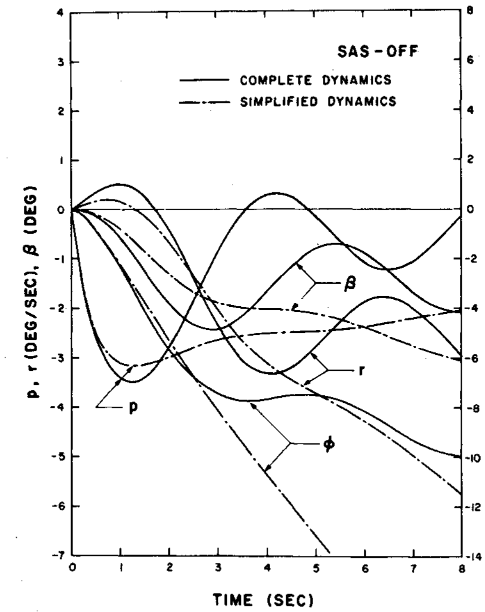

Fig. 6a SAS-off responses of aircraft 2 to a 3-deg step aileron input.

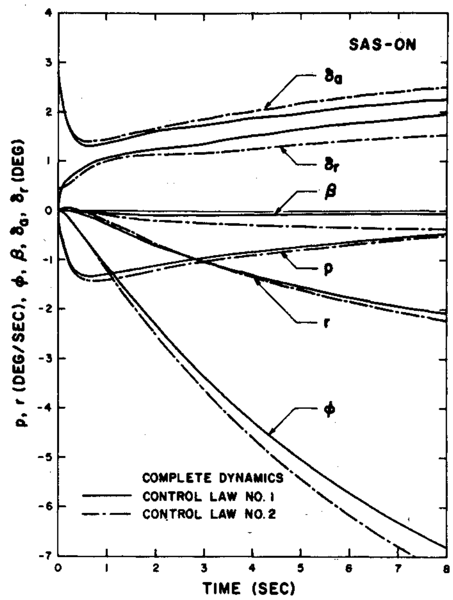

Fig. 7a SAS-on responses of aircraft 2 to a 3-deg step aileron input.

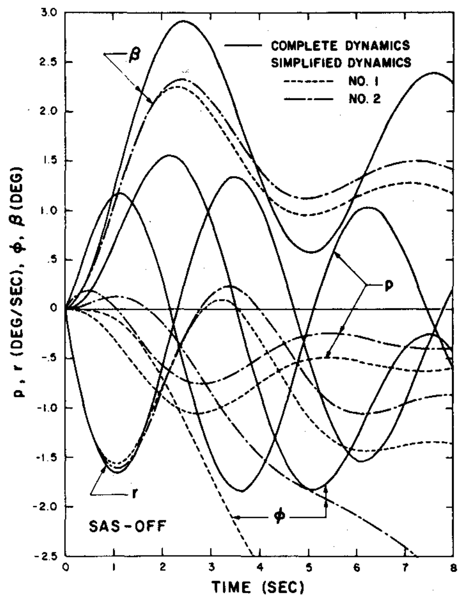

Fig. 6b SAS-off responses of aircraft 2 to a 3-deg step rudder input.

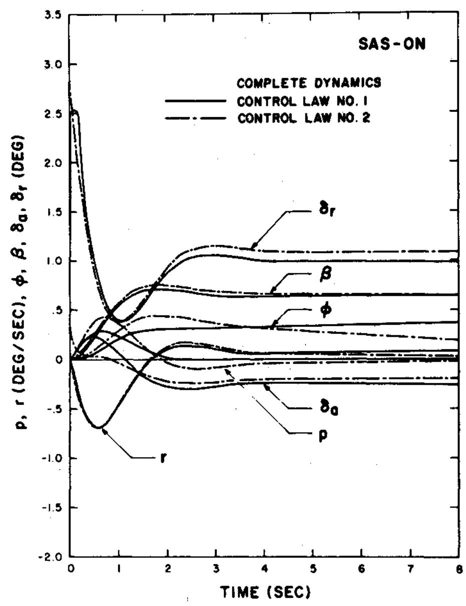

Fig. 7b SAS-on responses of aircraft 2 to a 3-deg step rudder input.

VII. Conclusions

The efficiency of the model-matching method for achieving desirable lateral handling qualities was illustrated in this paper. The decoupled control laws were derived analytically and are given in Eqs. (7) and (11). The feedback and feed-forward gains required for the lateral SAS design are given in terms of the stability derivatives and are calculated easily using these equations. Although a few of the stability derivatives of the aircraft dynamics were neglected during the design process, these control laws were substantiated for both conventional and STOL aircraft using simulation studies. Both control laws produced satisfactory responses. However, control law 2, which does not require any auxiliary dynamics, is simpler than control law 1.

Appendix A: Equicontrollable Canonical Form

The definition of equicontrollability in Sec. II shows that the first n columns of the controllability matrix $Q_c(n)$,

$$Q_c(n)=[B,\quad AB,\quad \ldots,\quad A^{n-1}B]$$

must be linearly independent in order for a system to be equicontrollable. This implies that each of the inputs of the system has to drive at least ρ integrators. It should be noted that any single-input system is equicontrollable. Generally, for multi-input systems $\rho \leq n$ and $m\rho \geq n$, and the condition imposed on the equicontrollability ($n=m\rho$) seems to require a substantial structural constraint to be placed on the system. All systems with minimal realization are not necessarily equicontrollable. For unequicontrollable systems, however, one can augment the system with some auxiliary states that do not modify the transfer function but alter the structure so as to achieve equicontrollability. Such states have to be controllable but are not observable. A procedure for the augmentation of a system is given by Curran.[12]

If the system $(\bar{A},\ \bar{B},\ \bar{C})$ is equicontrollable, there exists an $m \times m\rho$ matrix E such that

$$EQ=[I,\quad \theta,\quad \ldots,\quad \theta] \tag{A1}$$

where Q is defined as

$$Q=[\bar{A}^{\rho-1}\bar{B},\quad \ldots,\quad \bar{A}\bar{B},\quad \bar{B}] \tag{A2}$$

A similarity transformation T_I composed by E and $\bar{A}$,

$$T_I = \begin{bmatrix} E \\ E\bar{A} \\ \vdots \\ E\bar{A}^{\rho-1} \end{bmatrix} \tag{A3}$$

then can be used to transform $(\bar{A}, \bar{B}, \bar{C})$ into the system of equicontrollable canonical form (abbreviated as ECCF) $(\tilde{A}, \tilde{B}, \tilde{C}) = (T_I\bar{A}T_I^{-1}, T_I\bar{B}, \bar{C}T_I^{-1})$. The triplet $(\tilde{A}, \tilde{B}, \tilde{C})$ has the form given in Eqs. (3). For simplicity of notation, let the relationship of a coordinate transformation of matrices be denoted by

$$(\bar{A}, \bar{B}, \bar{C}) \xrightarrow{T_I} (\tilde{A}, \tilde{B}, \tilde{C})$$

The transfer function of the system $(\tilde{A}, \tilde{B}, \tilde{C})$ is represented by

$$W_p(\tilde{A}, \tilde{B}, \tilde{C}) = \tilde{C}(s,\rho)[s^\rho I - \tilde{A}(s,\rho)]^{-1} \tag{A4}$$

where

$$\tilde{A}(s,\rho) = \sum_{i=1}^{\rho} \tilde{A}_i s^{i-1} \qquad \tilde{C}(s,\rho) = \sum_{i=1}^{\rho} \tilde{C}_i s^{i-1}$$

Notations similar to the foregoing will be used for purposes of simplicity in Appendix B.

Appendix B: Model-Matching Algorithm

The model-matching algorithm used in this paper is described in the following, together with minor corrections of the original form given in Ref. 12. When the plant and model are expressed by Eqs. (1) and (2), the model-matching controller is designed by the following algorithm, which contains six steps with no iteration:

Step 1) The plant and model of Eqs. (1) and (2) are transformed into Luenberger's controllable canonical form.[14] It is defined by the transformation matrix T_L of the plant that

$$x_{pL} = T_L x_p \qquad (A, B, C) \xrightarrow{T_L} (A_L, B_L, C_L)$$

Luenberger's form shows the number of integrator which each input derives, and it corresponds to the dimension of each block on the diagonal blocks in the matrix A_L. If all of the blocks have the same dimension, the system is equicontrollable. This operation will be done in the next step.

Step 2) Find the smallest equicontrollable realization of the plant and model, and increase the number of states in the smaller so that they both have the same state dimension. In practical problems, the dimension of the plant usually is equal to or greater than that of the model. If the plant is not equicontrollable, an unobservable state vector, $z \epsilon R^{m\rho-n}$, with its poles at arbitrary location $-\alpha$, thus is added to the plant:

$$\dot{\bar{x}}_p = \bar{A}\bar{x}_p + \bar{B}u_p \qquad y_p = \bar{C}\bar{x}_p \tag{B1}$$

where

$$\bar{x}_p = \begin{bmatrix} x_{pL} \\ \hline z \end{bmatrix} \quad \bar{A} = \left[\begin{array}{c|c} A_L & 0 \\ \hline J_{12} & -\alpha I + J_{11} \end{array}\right] \quad \bar{B} = \begin{bmatrix} B_L \\ \hline 0 \end{bmatrix}$$

$$\bar{C} = [C_L \mid 0]$$

J_{11} and J_{12} can be found according to the definition of equicontrollability.

Similar manipulations are made for the model:

$$\dot{\bar{x}}_m = \bar{F}\bar{x}_m + \bar{G}u_m \qquad y_m = \bar{H}\bar{x}_m \tag{B2}$$

Here $\bar{x}_p$ and $\bar{x}_m \epsilon R^{m\rho}$, and ρ is the greater controllability index of the two.

Step 3) Equation (B1) is transformed by the matrix T_I into the ECCF $(\tilde{A}, \tilde{B}, \tilde{C})$, where $\tilde{x}_p = T_I\bar{x}_p$, and $(\bar{A}, \bar{B}, \bar{C}) \xrightarrow{T_I} (\tilde{A}, \tilde{B}, \tilde{C})$. The feedback

$$u_p = \tilde{K}\tilde{x}_p + \tilde{u}_p \qquad \tilde{K} = -[\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_\rho] \tag{B3}$$

then is applied to the plant, where $\tilde{u}_p$ is an input to the augmented plant, Eq. (B4). The result is

$$\dot{\tilde{x}}_p = (\tilde{A} + \tilde{B}\tilde{K})\tilde{x}_p + \tilde{B}\tilde{u}_p \qquad y_p = \tilde{C}\tilde{x}_p \tag{B4}$$

The model, Eq. (B2), also is transformed into ECCF:

$$\dot{\tilde{x}}_m = \tilde{F}\tilde{x}_m + \tilde{G}u_m \qquad y_m = \tilde{H}\tilde{x}_m \tag{B5}$$

Step 4) Find $L_i \epsilon R^{m \times m}$ and scalars λ_i $(i = 1, \ldots, \mu, \lambda_\mu = 1)$ from the equation

$$\frac{L(s,\mu)}{\lambda(s,\mu)} = \frac{\beta^\nu}{(s+\beta)^\nu} \tilde{C}(s,\rho)^{-1}\tilde{H}(s,\rho) \tag{B6}$$

where μ is an appropriate positive integer, and β is a positive constant at the designer's disposal. The factor $\beta^\nu/(s+\beta)^\nu$ on the right-hand side of the equation guarantees the realizability of the compensator, and ν is determined so that the degree of the denominator of the right-hand side is no smaller than that of the numerator. If $\nu \neq 0$, the transfer function of the model means to be modified by the factor.

Using the matrices L_i, the dynamic compensation with the state $\tilde{z} \epsilon R^{m(\mu-1)}$ is applied to the plant:

$$\dot{\hat{x}}_p = \hat{A}\hat{x}_p + \hat{B}\hat{u}_p \qquad y_p = \hat{C}\hat{x}_p$$

$$W_p(\hat{A}, \hat{B}, \hat{C}) = \tilde{C}(s,\rho)L(s,\mu)/s^{\rho^*} \tag{B7}$$

where $\rho^* \triangleq \rho + \mu - 1$, and

$$\hat{x}_p = \begin{bmatrix} \tilde{x}_p \\ \hline \tilde{z} \end{bmatrix} \qquad \hat{A} = \left[\begin{array}{c|c} \tilde{A} + \tilde{B}\tilde{K} & \tilde{B}J_{21} \\ \hline 0 & J_{22} \end{array}\right]$$

$$\hat{B} = \begin{bmatrix} \tilde{B}L_\mu \\ \hline \tilde{L} \end{bmatrix} \qquad \hat{C} = [\tilde{C} \mid 0]$$

$$J_{21} = [I, 0, \ldots, 0] \quad (\mu - 1) \text{ blocks}$$

$$J_{22} = \begin{bmatrix} 0 & I & \ldots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \ldots & I \\ 0 & 0 & \ldots & 0 \end{bmatrix} \qquad \tilde{L} = \begin{bmatrix} L_{\mu-1} \\ \vdots \\ L_2 \\ L_1 \end{bmatrix}$$

$$(\mu - 1) \text{ blocks square} \tag{B8}$$

Step 5) Equations (B7) are transformed into the ECCF by the similarity matrix $T_2 \epsilon R^{m\rho^* \times m\rho^*}$:

$$T_2^{-1} = \begin{bmatrix} L_1 & L_2 & \ldots & L_\mu & 0 & \ldots & 0 \\ 0 & L_1 & \ldots & L_{\mu-1} & L_\mu & \ldots & 0 \\ \vdots & \vdots & & & & & \vdots \\ 0 & 0 & \ldots & \ldots & \ldots & \ldots & L_1 \end{bmatrix} \tag{B9}$$

It is defined that $\hat{x}_p^* = T_2 \hat{x}_p$, and $(\hat{A}, \hat{B}, \hat{C}) \overset{T_2}{\to} (\hat{A}^*, \hat{B}^*, \hat{C}^*)$. Then the following feedback is applied to the plant:

$$\hat{u}_p = \hat{K}^* \hat{x}_p^* + u_m \qquad \hat{K}^* \triangleq \hat{K} T_2^{-1} \tag{B10}$$

where the block elements $\hat{K}_i^* \in R^{m \times m}$ $(i=1, \ldots, \rho^*)$ of the matrix $\hat{K}^*$ can be determined by the equation

$$\hat{K}^*(s, \rho^*) = s^{\rho^*} I - [s^\rho I - \bar{F}(s,\rho)] \lambda(s,\mu) \tag{B11}$$

Step 6) Transform the compensator so far derived to the original coordinate, and find a minimal realization[13] of it. The result is that

$$u_p = Sz + N\bar{z} + Qx_p + L_\mu u_m \tag{B12a}$$

$$\dot{z} = (-\alpha I + J_{11}) z + J_{12} T_L x_p \tag{B12b}$$

$$\dot{\bar{z}} = Rz + M\bar{z} + Px_p + \bar{L} u_m \tag{B12c}$$

where

$$S = [(\tilde{K} + L_\mu \hat{K}_x) T_1]_z \qquad N = L_\mu \tilde{K}_z + J_{21}$$

$$Q = [(\tilde{K} + L_\mu \hat{K}_x) T_1]_x T_L \qquad M = \bar{L} \hat{K}_z + J_{22}$$

$$P = [\bar{L} \hat{K}_x T_1]_x T_L \qquad R = [\bar{L} \hat{K}_x T_1]_z \tag{B13}$$

and subscripts x and z denote decomposition of the matrix to their corresponding dimensions. Equations (B12) are the control law of the model-matching controller, and a block diagram for them is given in Fig. 1.

Acknowledgment

The work reported in this paper was supported by the National Council of Canada under Grants No. A-1080 and A5625.

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